Benha University
Faculty Of Engineering at Shoubra

ECE 122
Electrical Circuits (2)(2016/2017)
Lecture (12)
Three Phase Systems

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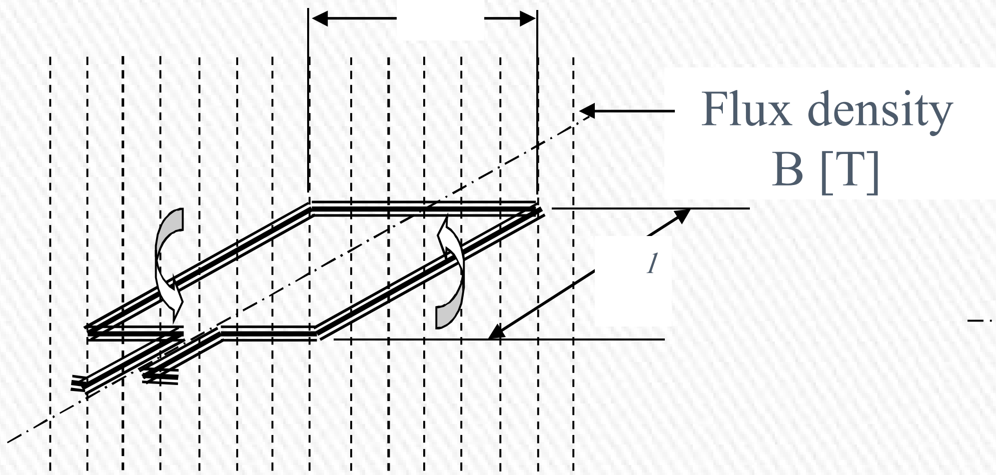
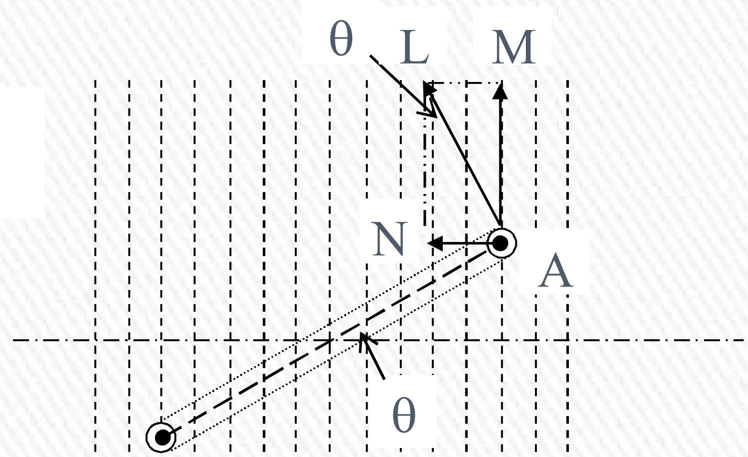
WHY WE STUDY 3 PHASE SYSTEM ?

- ALL electric power system in the world used 3-phase system to GENERATE, TRANSMIT and DISTRIBUTE
  - ✓ One phase, two phase, or three phase can be taken from three phase system rather than generated independently.
- Instantaneous power is constant (not pulsating).– thus smoother rotation of electrical machines
  - ✓ High power motors prefer a steady torque
- More economical than single phase – less wire for the same power transfer
  - ✓ The amount of wire required for a three phase system is less than required for an equivalent single phase system.

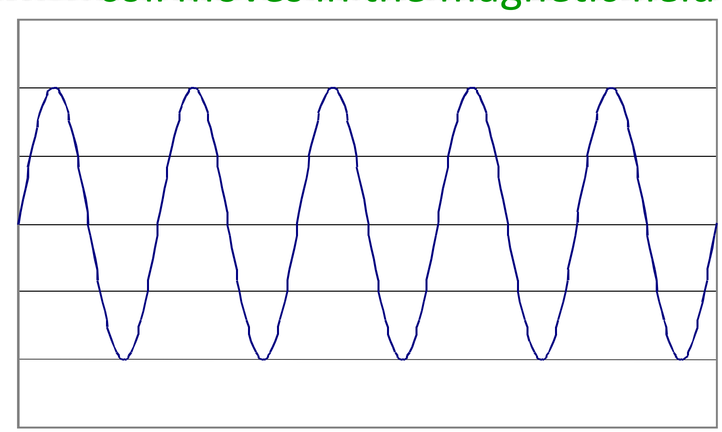
Single phase generator



Generator for single phase



Current induces in the coil as the coil moves in the magnetic field

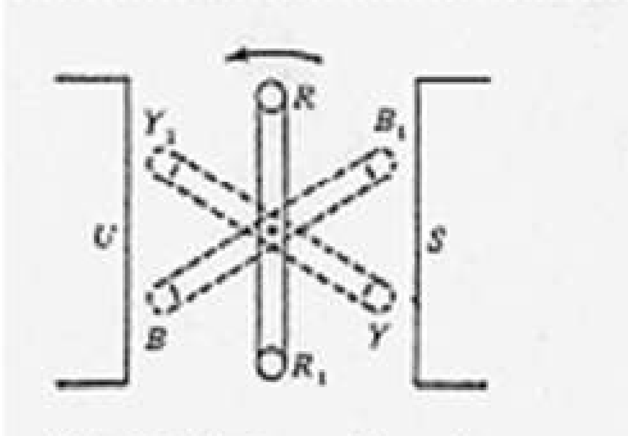


Current produced at terminal

Note

Induction motor cannot start by itself. This problem is solved by introducing three phase system

Three phase generator



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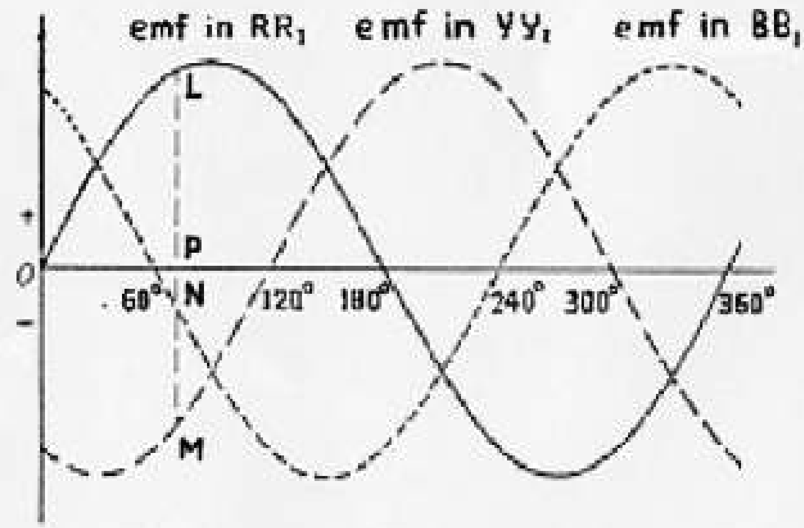
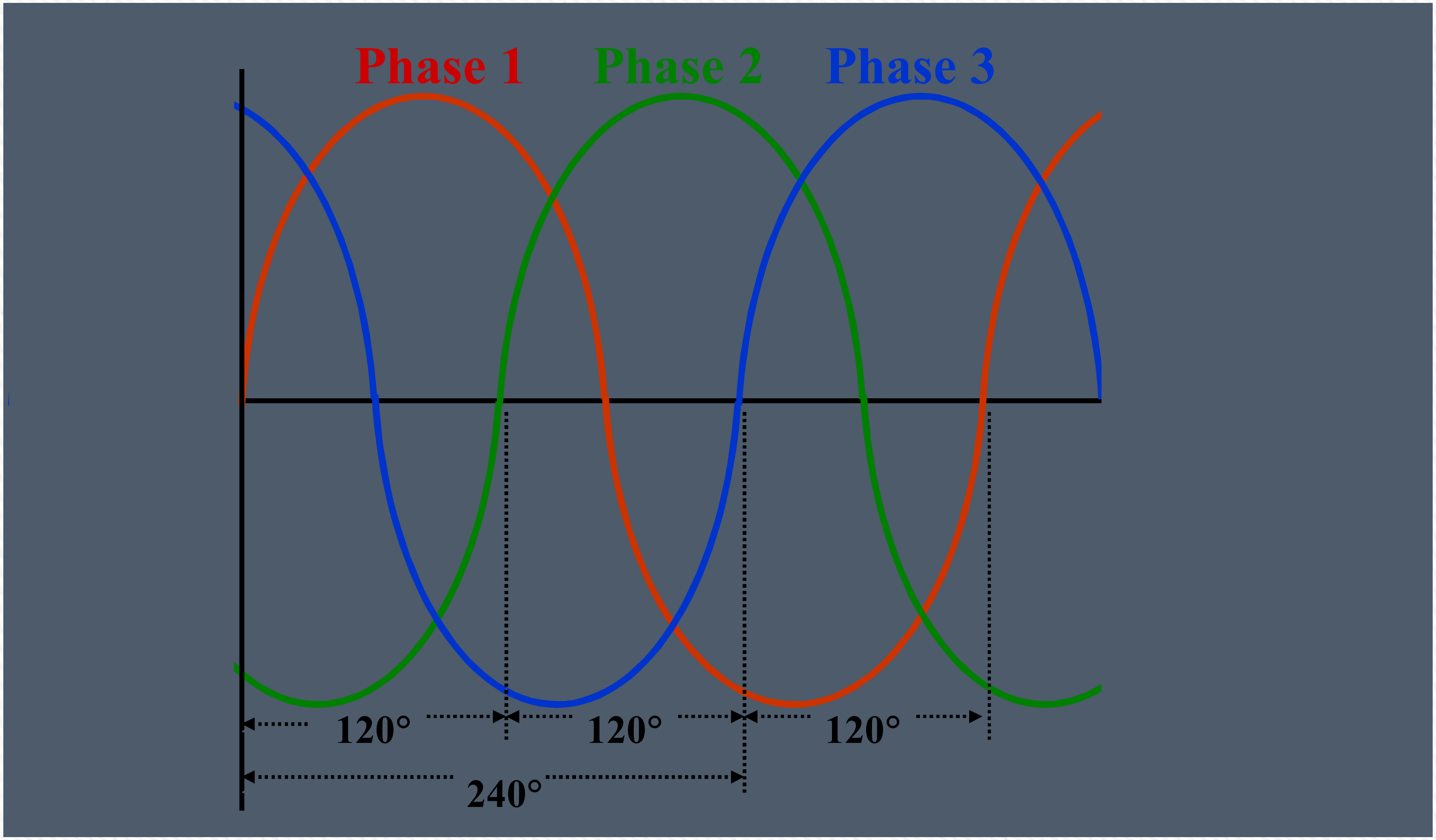


Instead of using one coil only , three coils are used arranged in one axis with orientation of 120° each other. The coils are $R\text{-}R_1$, $Y\text{-}Y_1$ and $B\text{-}B_1$. The phases are measured in this sequence R-Y-B. I.e Y lags R by 120° , B lags Y by 120°.

THREE-PHASE WAVEFORM

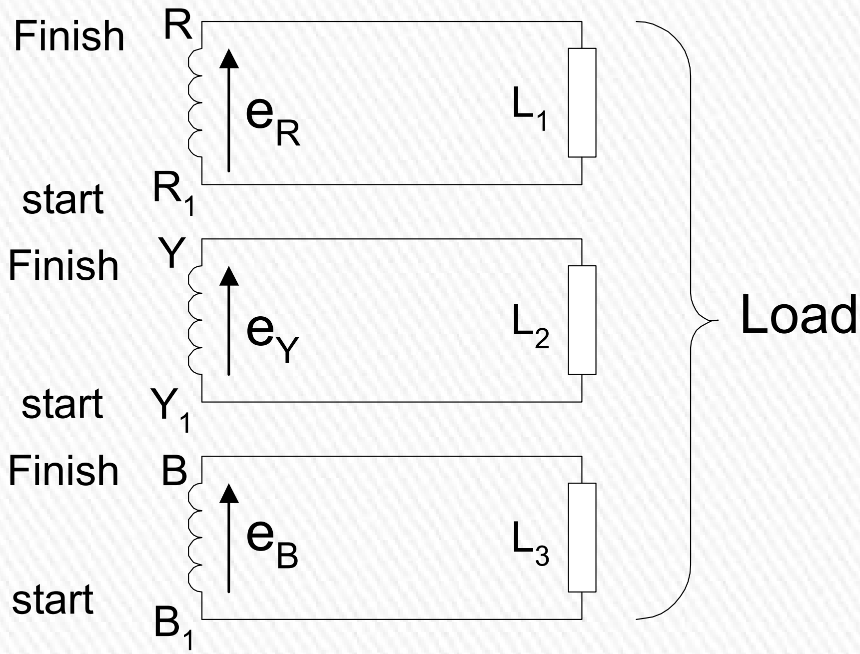


Phase 2 lags phase 1 by 120°.

Phase 3 lags phase 1 by 240°.

Phase 2 leads phase 3 by 120°.

Phase 1 leads phase 3 by 240°.



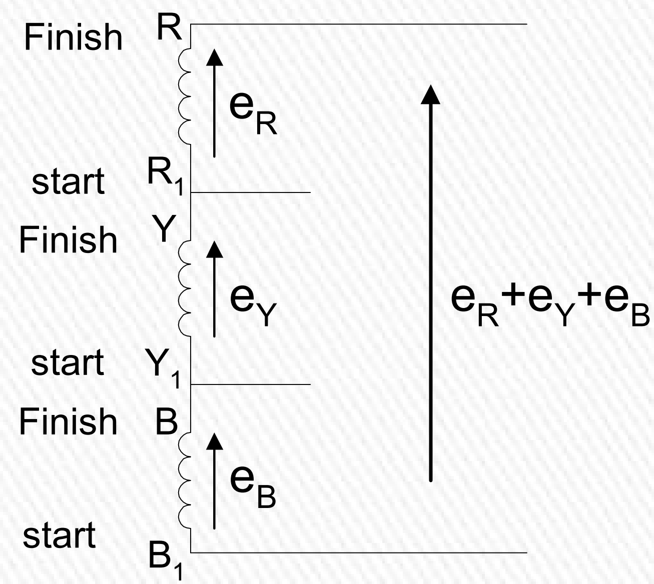
The three winding can be represented by the above circuit. In this case we have six wires. The emf are represented by e_R , e_Y, e_B.

$$e_R = E_m \sin \omega t$$

$$e_Y = E_m \sin(\omega t\text{-}120^\circ)$$

$$e_B = E_m \sin(\omega t\text{-}240^\circ)$$

The circuit can be simplified as follows, where R_1 can be connected to Y and Y_1 can be connected to B. In this case the circuit is reduced to 4 wires.



$$e_{RB1} = e_R + e_Y + e_B$$

$$= E_m[\sin \omega t + \sin(\omega t - 120^\circ) + \sin(\omega t - 240^\circ)]$$

$$= E_m[\sin \omega t + \sin \omega t \cdot \cos 120^\circ - \cos \omega t \cdot \sin 120^\circ$$

$$+ \sin \omega t \cdot \cos 240^\circ - \cos \omega t \cdot \sin 240^\circ]$$

$$= E_m[\sin \omega t - 0.5 \sin \omega t - 0.866 \cos \omega t - 0.5 \sin \omega t + 0.866 kos\, \omega t]$$

$$= 0$$

Since the total emf is zero, R and B_1 can be connected together, thus we arrive with delta connection system.

Delta connection of three phase windings

Fig.A

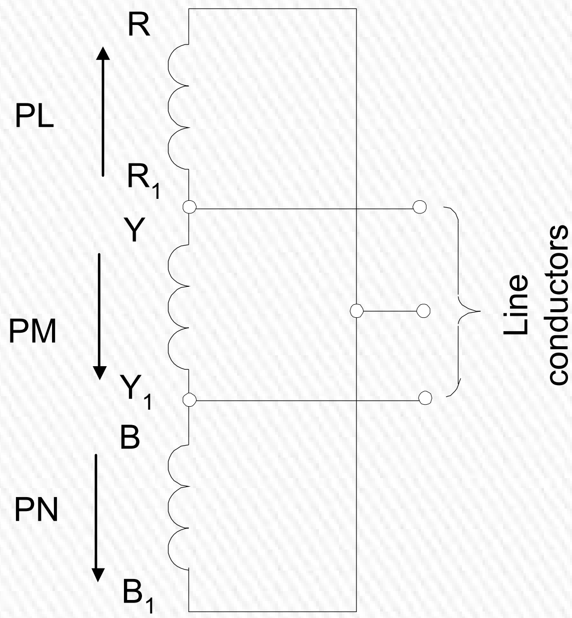


Fig. B

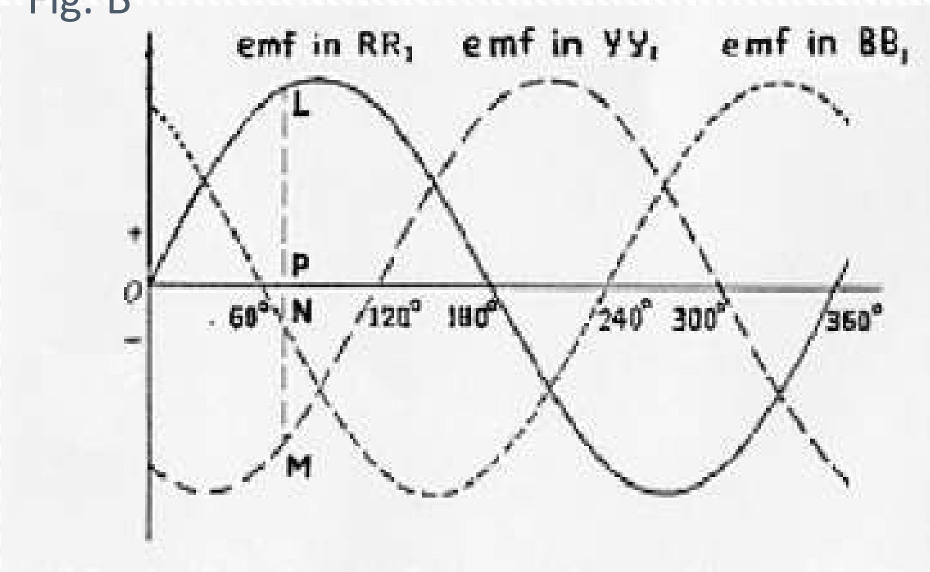
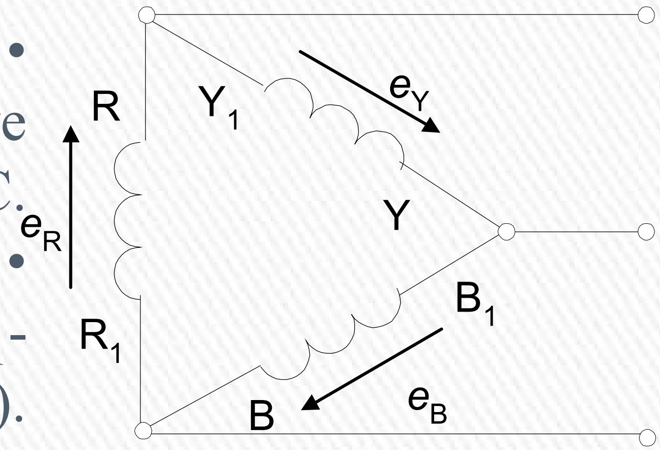


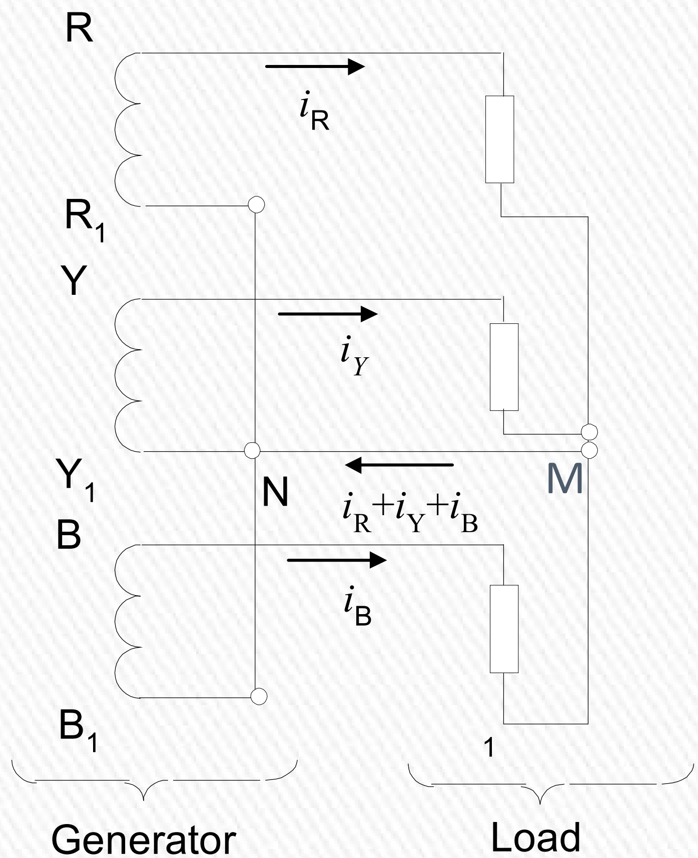
Fig. C



- Since the total emf is zero, R and B_1 can be connected together as in Fig.A , thus we arrive with delta connection system as in Fig. C.
- The direction of the emf can be referred to the emf waveform as in Fig. B where PL is +ve (R_1-R), PM is –ve (Y-Y_1) and PN is –ve (B-B_1).

Star connection of three phase windings

- R_1, Y_1 and B_1 are connected together.
- As the e.m.f generated are assumed in positive direction , therefore the current directions are also considered as flowing in the positive direction.
- The current in the common wire (MN) is equal to the sum of the generated currents. i.e $i_R+i_Y+i_B$.
- This arrangement is called **four –wire star-connected system**. The point N refers to star point or neutral point.



The instantaneous current in loads L_1 , L_2 and L_3 are

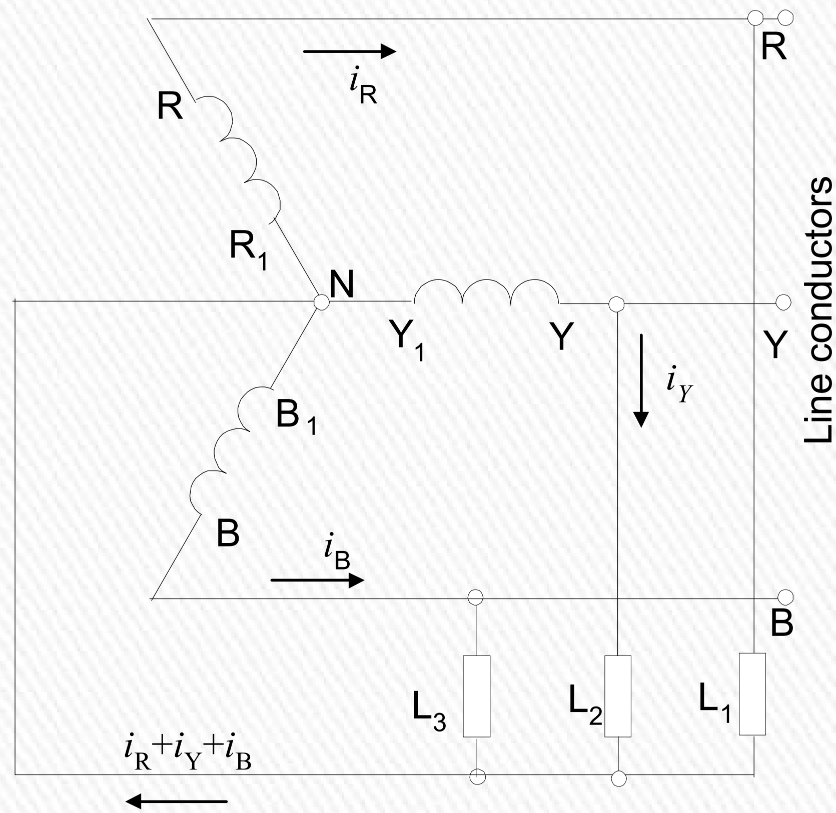
$$i_R = I_m \sin \omega t$$

$$i_Y = I_m \sin(\omega t\text{-}120^\circ)$$

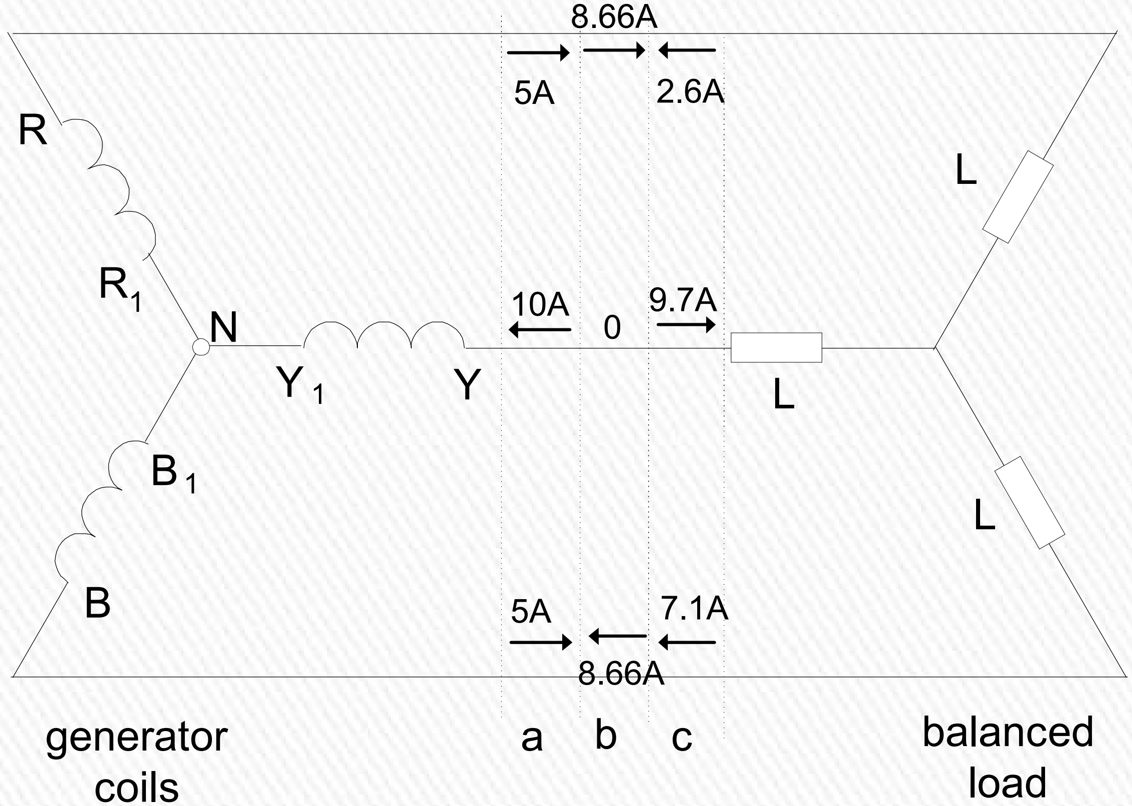
$$i_B = I_m \sin(\omega t\text{-}240^\circ)$$

$$i_N = i_R + i_Y + i_B$$

$$= I_m[\sin \omega t + \sin(\omega t - 120^\circ) + \sin(\omega t - 240^\circ)] = 0$$

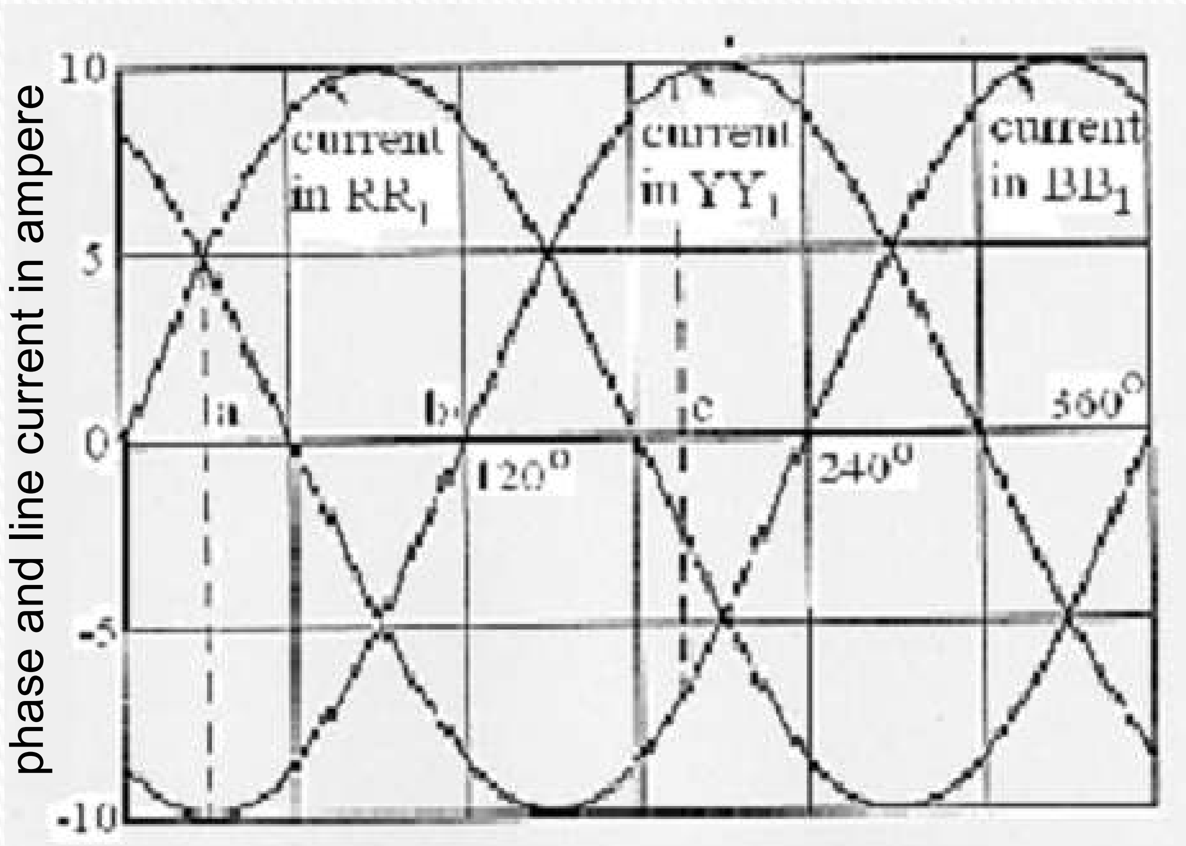


For balanced loads, the fourth wire carries no current , so it can be dispensed



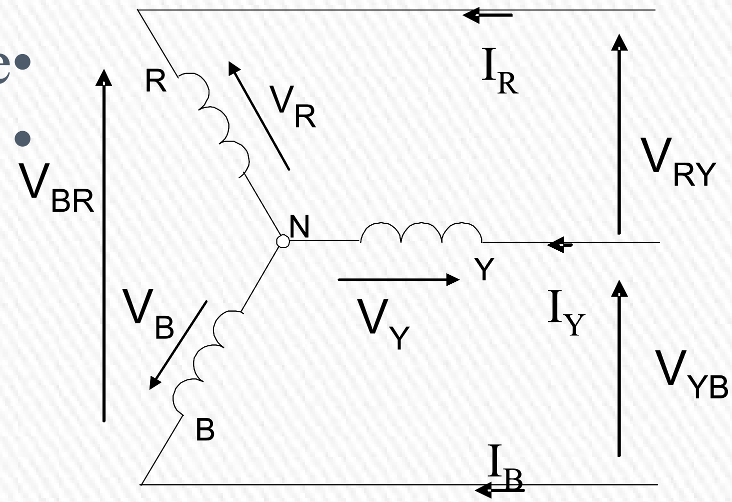
Three-wire star-connected system with balanced load

Instantaneous currents' waveform for i_R, i_Y and i_B in a balanced three-phase system.



Voltage and current in star connection

- V_{RY}, V_{YB} and V_{BR} are called line voltage
- V_R, V_Y and V_B are called phase voltage



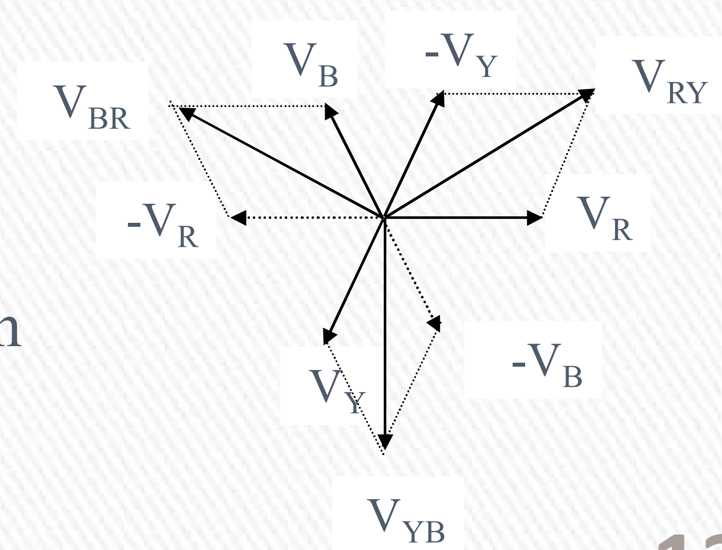
From Kirchoff voltage law we have

$$V_{RY} = V_R - V_Y = V_R + (-V_Y)$$

$$V_{YB} = V_Y - V_B = V_Y + (-V_B)$$

$$V_{BR} = V_B - V_R = V_B + (-V_R)$$

In phasor diagram

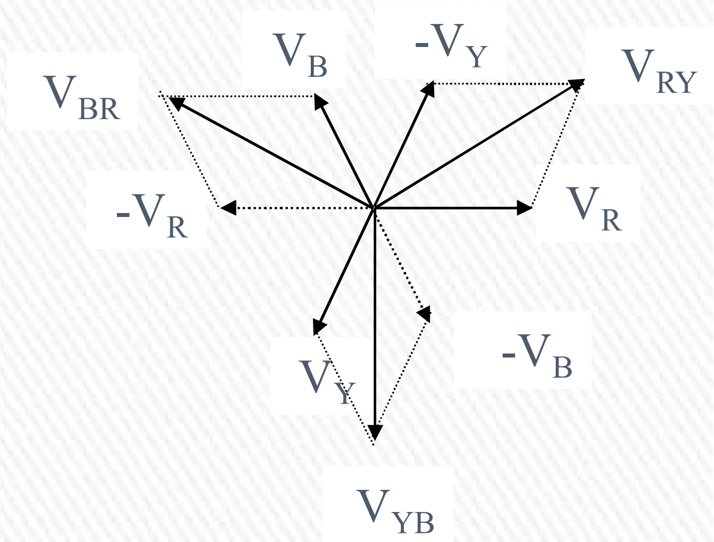


For balanced load V_R , V_Y and V_B are equaled but out of phase

$V_R = V_P\angle 30°;$ $V_{RY} = V_L\angle 30°;$

$V_Y = V_P\angle -90°;$ $V_{YB} = V_L\angle -90°;$

$V_B = V_P\angle 150°;$ $V_{BR} = V_L\angle 150°;$



therefore

$$V_{RY} = 2V_R \cos 30^o = \left(\sqrt{3}\right)V_P$$

$$V_{BR} = 2V_B \cos 30^o = \left(\sqrt{3}\right)V_P$$

$$V_{YB} = 2V_Y \cos 30^o = \left(\sqrt{3}\right)V_P$$

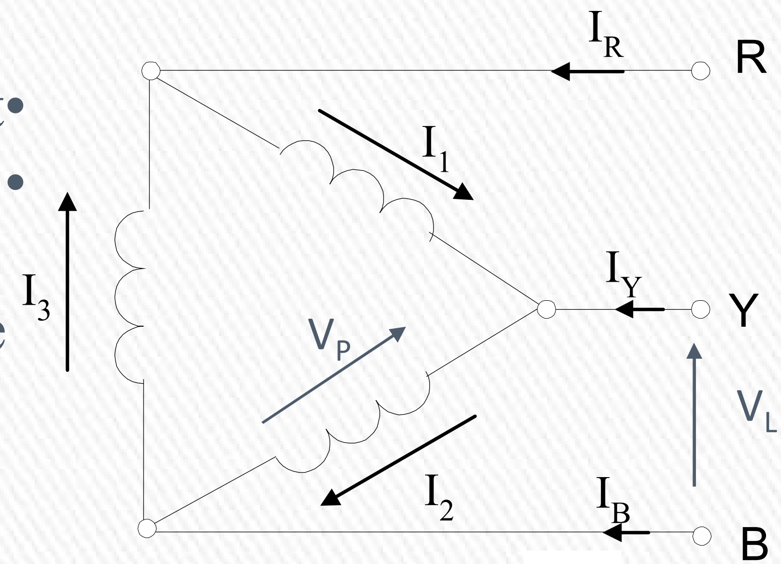
then

$\sqrt{}$

and

Voltage and current in Delta connection

- I_R, I_Y and I_B are called line current
- I_1, I_2 and I_3 are called phase current



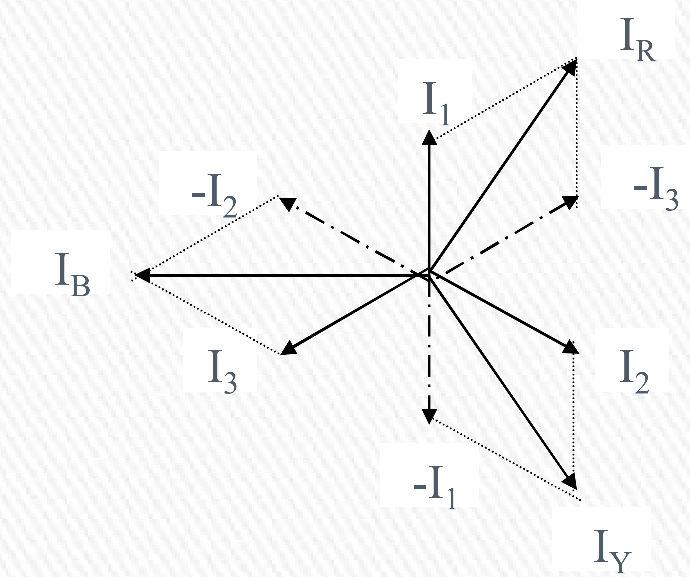
From Kirchoff current law we have

$$I_R = I_1 - I_3 = I_1 + (-I_3)$$

$$I_Y = I_2 - I_1 = I_2 + (-I_1)$$

$$I_B = I_3 - I_2 = I_3 + (-I_2)$$

In phasor diagram



Since the loads are balanced, the magnitude of currents are equaled but 120o out of phase. i.e $I_1 = I_2 = I_3 , = I_P$ Therefore:-

$I_R = I_L\angle 30°$; $I_1 = V_P\angle 30°$;

$I_Y = I_L\angle -90°$; $I_2 = V_P\angle -90°$;

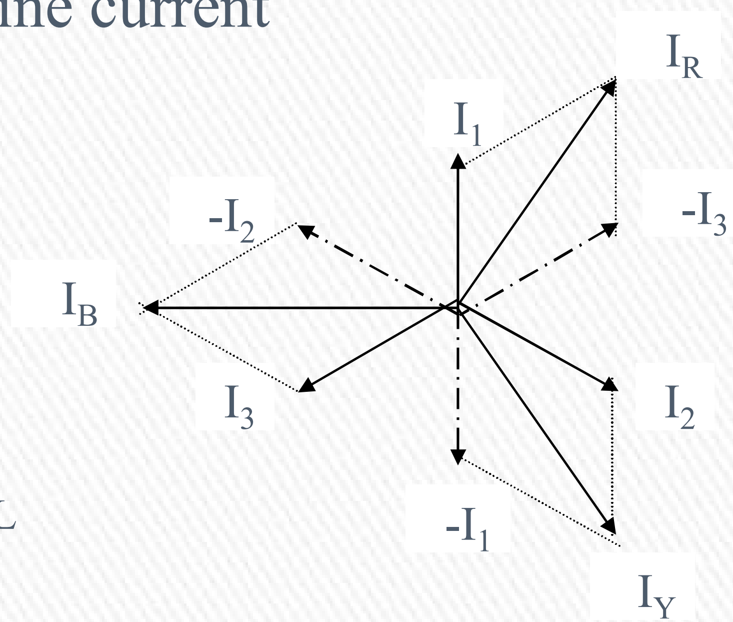
$I_B = I_L\angle 150°$; $I_3 = V_P\angle 150°$;

Where I_P is a phase current and I_L is a line current

$$I_R = 2I_1 \cos 30^\circ = (\sqrt{3})I_P$$

$$I_Y = 2I_2 \cos 30^\circ = (\sqrt{3})I_P$$

$$I_B = 2I_3 \cos 30^\circ = \left(\sqrt{3}\right)I_P$$



Thus $I_R = I_Y = I_B = I_L$

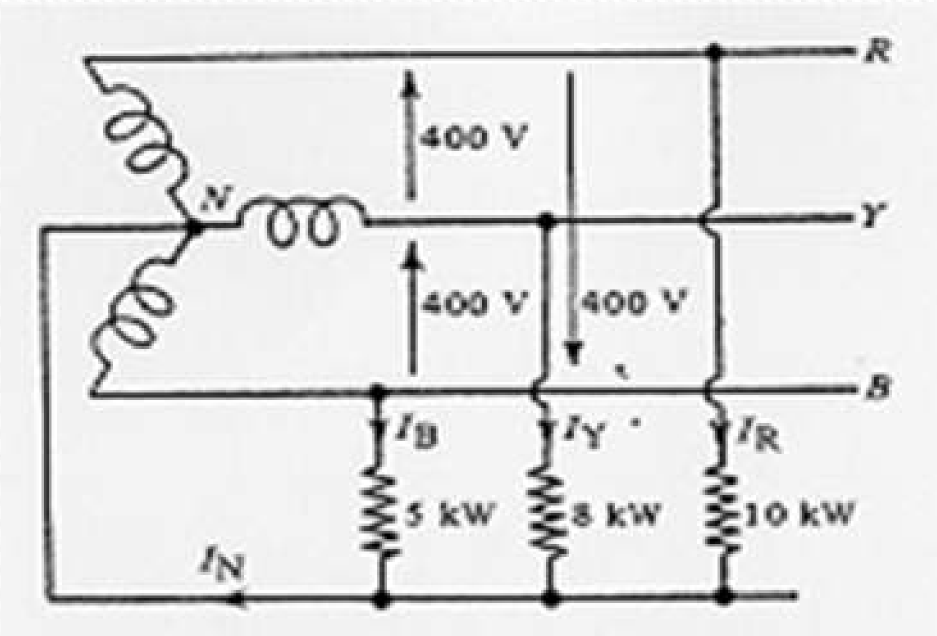
Hence

$$\sqrt{}$$

Unbalanced load

Example 1

In a three-phase four-wire system the line voltage is 400V and non-inductive loads of 5 kW, 8 kW and 10 kW are connected between the three conductors and the neutral. Calculate: (a) the current in each phase (b) the current in the neutral conductor.

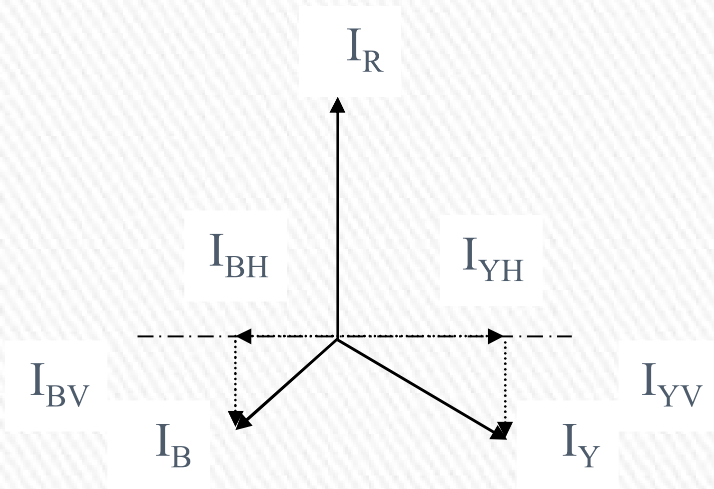


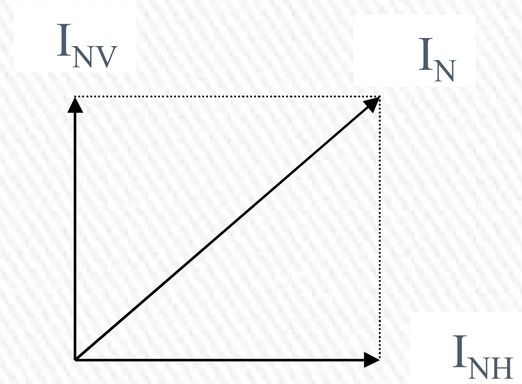
Voltage to neutral $$V_P = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230V$$

Current in 10kW resistor $$I_R = \frac{P_R}{V_P} = \frac{10^4}{230} = 43.5A$$

Current in 8kW resistor $$I_Y = \frac{P_Y}{V_P} = \frac{8\times10^3}{230} = 34.8A$$

Current in 5kW resistor $$I_B = \frac{P_B}{V_P} = \frac{5\times10^3}{230} = 21.7A$$





Resolve the current components into horizontal and vertical components.

$$I_H = I_Y \cos 30^\circ - I_B \cos 30^o = 0.866(34.8 - 21.7) = 11.3A$$

$$I_V = I_R - I_Y \cos 60^\circ - I_B \cos 60^o = 43.5 - 0.5(34.8 + 21.7) = 13.0A$$

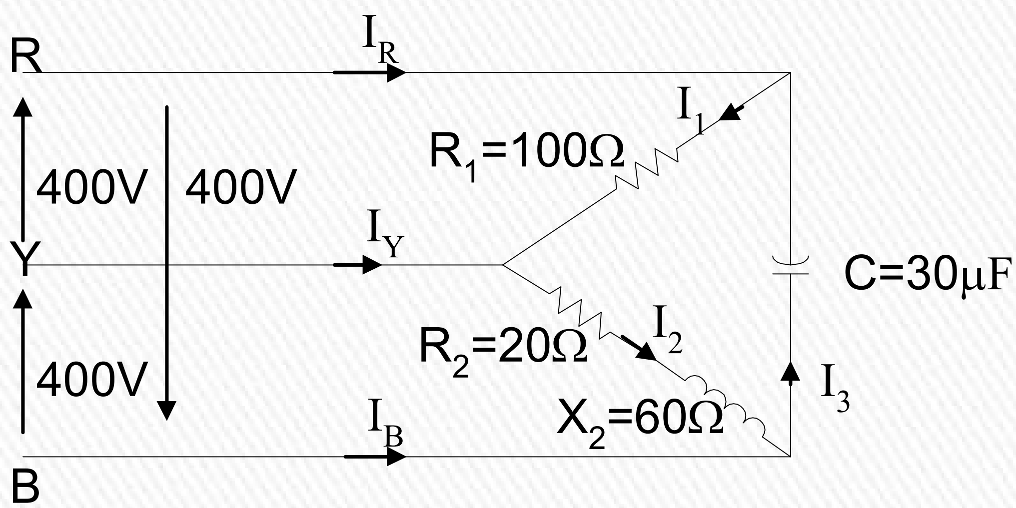
$$I_N = \sqrt{I_{NH}{}^2 + I_{NV}{}^2} = \sqrt{11.3^2 + 13.0^2} = 17.2A$$

Example 2

A delta –connected load is arranged as in Figure below.
The supply voltage is 400V at 50Hz. Calculate:

(a)The phase currents;

(b)The line currents.



(a)

$$I_1 = \frac{V_{RY}}{R_1} = \frac{400}{100} = 4A$$

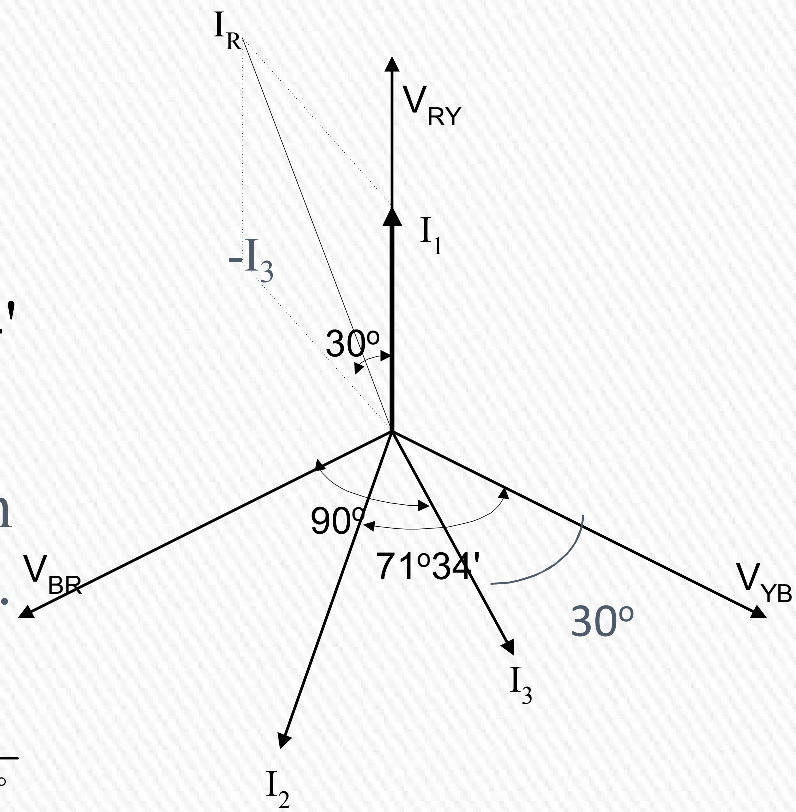
I_1 is in phase with V_{RY} since there is only resistor in the branch

In branch between YB , there are two components , R_2 and X_2

$$I_2 = \frac{V_{YB}}{Z_Y} = \frac{400}{\sqrt{20^2 + 60^2}} = 6.32A$$

$$Z_Y = \sqrt{R_2{}^2 + X_2{}^2} \qquad = \sqrt{20^2 + 60^2}$$

$$\theta_Y = \tan^{-1}\left(\frac{X_2}{R_2}\right) = \tan^{-1}\left(\frac{60}{20}\right) = 71^\circ 34'$$



In the branch RB , only capacitor in it , so the X_C is -90 out of phase.

$$I_3 = \frac{V_{BR}}{X_C} \qquad = \frac{400}{1/(2\pi \times 50 \times 30 \times 10^{-6})\angle -90^\circ}$$

$$= 3.77A \angle 90^o$$

(b) $I_R = I_1 - I_3$

$$I_R^2 = I_1^2 + 2I_1I_3\cos\theta + I_3^2$$ $\theta = 30^o$

$$I_R^2 = (4.0)^2 + 2(4.0)(3.77)\cos 30^o + (3.77)^2 = 56.3$$

$$I_R = 7.5A$$

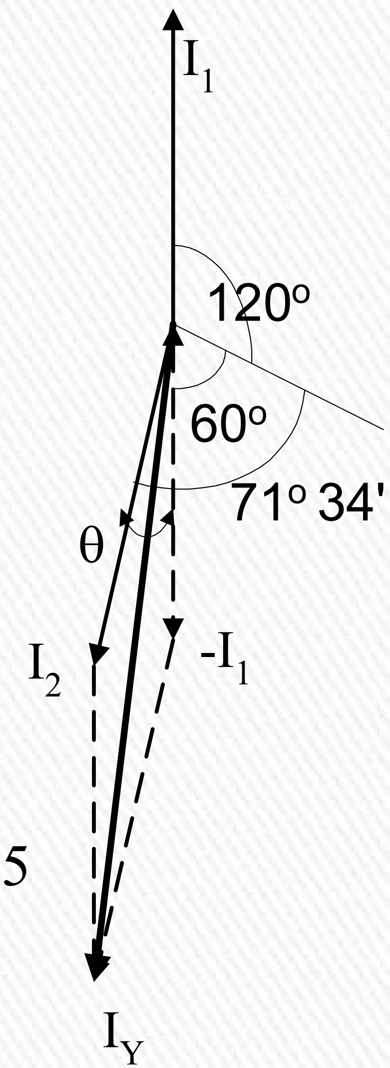
$\theta = 71^o\ 34' - 60^o = 11^o\ 34'$

$I_Y = I_2 - I_1$

$$I_Y^2 = I_2^2 + 2I_1I_2\cos\theta + I_1^2$$

$$I_Y^2 = (6.32)^2 + 2(4.0)(6.32)\cos 11^o 34' + (4.0)^2 = 105.5$$

$$I_Y = 10.3A$$



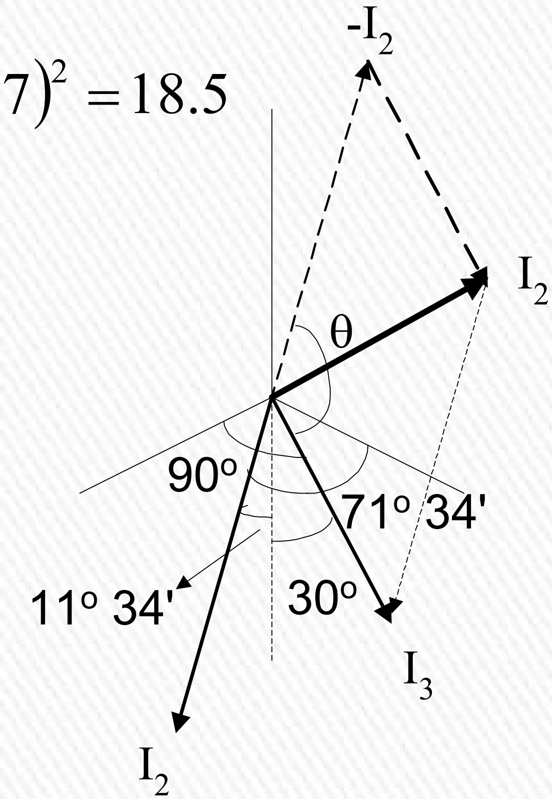
$\theta = 180\text{-}30^o\text{-}11^o\ 34' = 138^o\ 34'$

$$I_B = I_3 - I_2$$

$$I_B^2 = I_3^2 + 2I_3I_2\cos\theta + I_2^2$$

$$I_B^2 = (6.32)^2 + 2(3.77)(6.32)\cos 138^o 26' + (3.77)^2 = 18.5$$

$$I_B = 4.3A$$



Power in three phase

Active power per phase = $I_P V_P$ x power factor
Total active power= 3 $V_P I_P$ x power factor

$$P = 3V_P I_P \cos\varphi$$

If I_L and V_L are rms values for line current and line voltage respectively. Then for delta (Δ) connection: $V_P = V_L$ and $I_P = I_L/\sqrt{3}$. therefore:

$$P = \sqrt{3} V_L I_L \cos\varphi$$

For star connection (Y) : $V_P = V_L/\sqrt{3}$ and $I_P = I_L$. therefore:

$$P = \sqrt{3} V_L I_L \cos\varphi$$

Example 3

A three-phase motor operating off a 400V system is developing 20kW at an efficiency of 0.87 p.u and a power factor of 0.82. Calculate:

(a) The line current;

(b) The phase current if the windings are delta-connected.

(a) Since $Efficiency = \frac{output\ \ power\ in\ watts}{input\ \ power\ in\ watts}$

$$\eta = \frac{output\ \ power\ in\ watts}{\sqrt{3}\ I_L V_L \times p.f}$$

$$0.87 = \frac{20 \times 1000}{\sqrt{3} \times I_L \times 400 \times 0.82}$$

And line current $= I_L = 40.0A$

(b) For a delta-connected winding

$$Phase\ current = \frac{line\ current}{\sqrt{3}} = \frac{40.0}{\sqrt{3}} = 23.1A$$

Example 4

Three identical coils, each having a resistance of 20Ω and an inductance of 0.5 H connected in (a) star and (b) delta to a three phase supply of 400 V; 50 Hz. Calculate the current and the total power absorbed by both method of connections.

First of all calculating the impedance of the coils

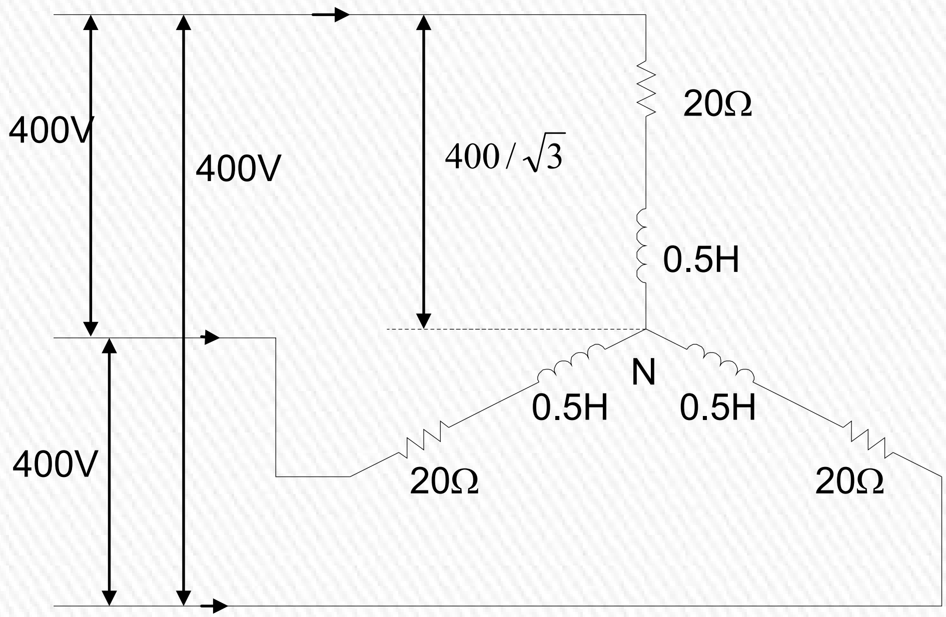
$$R_P = 20\Omega \qquad X_P = 2\pi \times 50 \times 0.5 = 157\Omega$$

$$Z_P = R_P + jX_P = \sqrt{R_P{}^2 + X_P{}^2}\angle\varphi \qquad \text{where} \quad \varphi = \tan^{-1}\left(\frac{X_P}{R_P}\right)$$

$$= \sqrt{20^2 + 157^2}\angle\tan^{-1}\left(\frac{157}{20}\right) = 158\angle 83^\circ$$

$$\cos\varphi = \cos 83^\circ = 0.1264$$

Star-connection



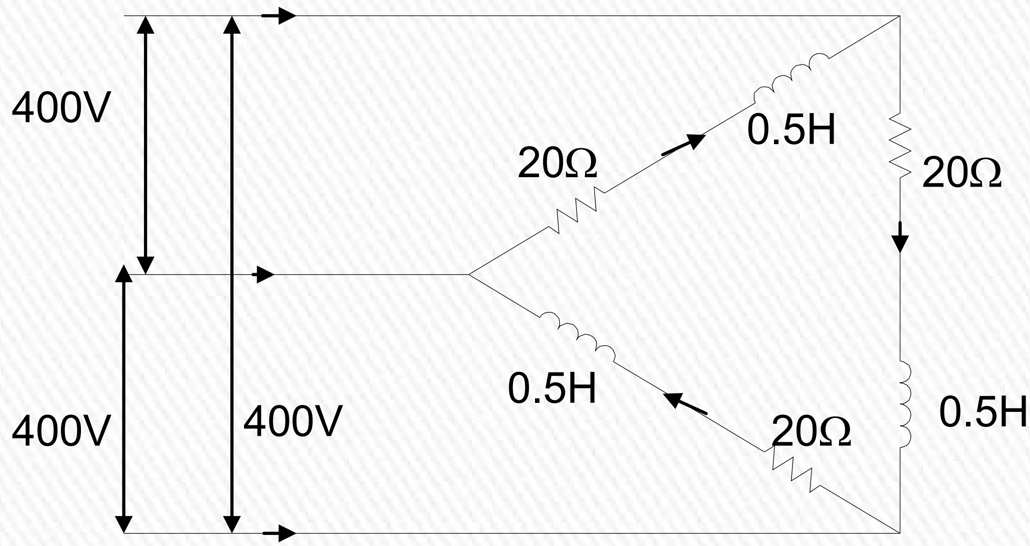
Since it is a balanced load

$$V_P = \frac{400}{\sqrt{3}} = 231\text{V} \qquad I_P = I_L = \frac{V_P}{Z_P} = \frac{231}{158} = 1.46\text{A}$$

Power absorbed

$$P = \sqrt{3} V_L I_L \cos\varphi \quad = \sqrt{3} \times 400 \times 1.46 \times 0.1264 = 128\text{W}$$

Star connection



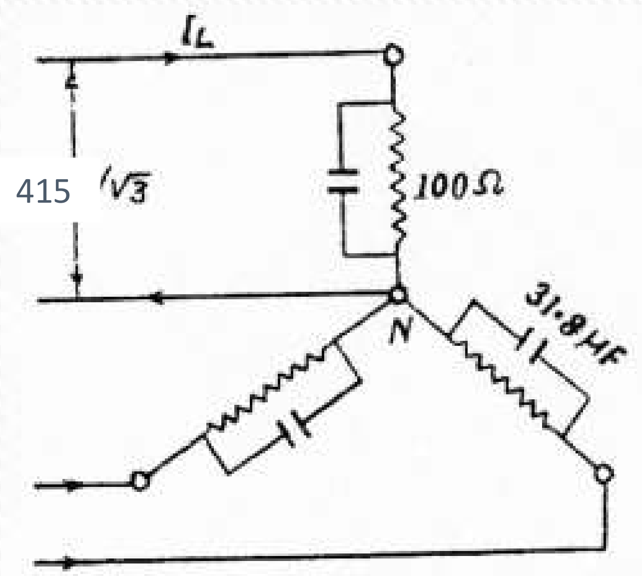
$$V_P = V_L = 400V \qquad I_P = \frac{V_P}{Z_P} = \frac{400}{158} = 4.38A$$

$$P = \sqrt{3} V_L I_L \cos\varphi = \sqrt{3} \times 400 \times 4.38 \times 0.1264 = 383W$$

Example 5

A balanced three phase load connected in star, each phase consists of resistance of 100 Ω paralleled with a capacitance of 31.8 μF. The load is connected to a three phase supply of 415 V; 50 Hz.

Calculate: (a) the line current;
(b) the power absorbed;
(c) total kVA;
(d) power factor .



$$V_P = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 240V$$

Admittance of the load

$$Y_P = \frac{1}{R_P} + \frac{1}{X_P} \qquad \text{where} \qquad X_P = \frac{1}{j\omega C}$$

$$= \frac{1}{R_P} + j\omega C \quad = \frac{1}{100} + j2\pi \times 50 \times 31.8 \times 10^{-6} \quad = (0.01 + j0.01)\text{S}$$

Line current

$$I_L = I_P = V_P Y_P = 240(0.01 + j0.01) \; = 2.4 + j2.4 = 3.39\angle 45^\circ$$

Volt-ampere per phase

$$P_{VA} = V_P I_P \qquad = 240 \times 3.39\angle 45^\circ = 814.4\angle 45^\circ$$

Active power per phase $\qquad P_{PA} = 814.4 \cos 45^\circ = 576$

Total active power $\qquad P_A = 3 \times 576 = 1.728\text{kW}$

(b)

Reactive power per phase $P_{PR} = j814.4\sin 45^{\circ} = j576$

Total reactive power $P_{R} = j3 \times 576 = j1.728\text{kW}$

(c) Total volt-ampere $= 3 \times 814.4 = 2.44\text{kVA}$

(d) Power Factor = cosφ = cos 45° = 0.707 (leading)

Example 6

A three phase star-connected system having a phase voltage of 230V and loads consist of non reactive resistance of 4 Ω, 5 Ω and 6Ω respectively.

Calculate:(a) the current in each phase conductor

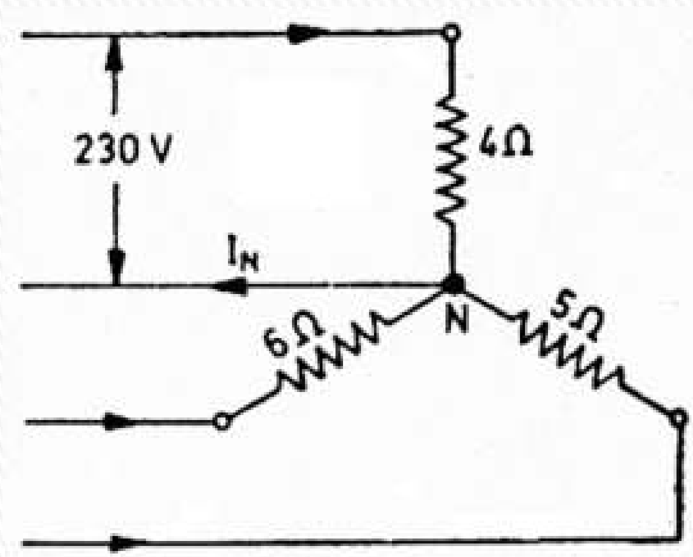
(b) the current in neutral conductor

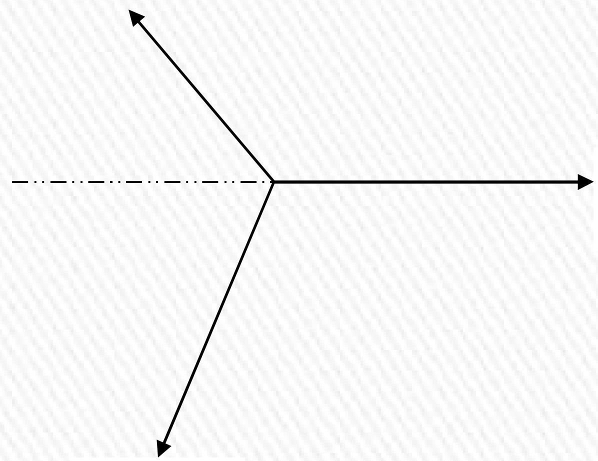
and (c) total power absorbed.

$$I_{4\Omega} = \frac{230}{4} = 57.5A$$

$$I_{5\Omega} = \frac{230}{5} = 46A$$

$$I_{6\Omega} = \frac{230}{6} = 38.3A$$





(b)

X-component = 46 cos 30° + 38.3 cos 30° - 57.5 = 15.5 A

Y-component = 46 sin 30° - 38.3 sin 30° = 3.9 A

Therefore $$I_N = \sqrt{15.5^2 + 3.9^2} = 16A$$

(c) $$P = 230(57.5 + 46 + 38.3) = 32.61kW$$


Thank You